2 L.C.M & H.C.F

L.C.M

Least Common Multiple (L.C.M) of numbers is defined as the least number, which is exactly divisible by each one of the given numbers. Remember, L.C.M. is always greater than or equal to the greatest of the given numbers. Also, there will be nos. greater than the L.C.M. but it will be the lowest of them all.

H.C.F or G.C.D

Highest Common Factor or ***Greatest Common Divisor*** of two or more numbers is the greatest number that divides each one of them exactly. H.C.F. denotes the highest possible factor that will divide each of the other nos. in a group or in other words there will be factors smaller than it which will also divide the same set of numbers. but H.C.F. will be the largest possible.

Methods of finding L.C.M

Methods of finding L.C.M

Factorisation Method:

Resolve each one of the given numbers into prime factors. Then the product of the highest power of all the factors gives the L.C.M.

Find the L.C.M of 2, 4, 6, 8, 10.

Factorize all the numbers into their prime factors 2, 2^2, 2×3, 2^3, 2×5 and choose the factors with highest power. Now, L.C.M. = $2^3 \times 3 \times 5 = 8 \times 3 \times 5$ = 120. Again recollect that there are other possibilities like 240, 360, 480, …or any other multiple of 120 but LCM is the smallest.

Alternative (shortcut) Method

2	2-4-6-8-10
2	1-2-3-4-5
	1-1-3-2-5

Now L.C.M = 2 x 2 x 3 x 2 x 5 = 120.

Methods of finding H.C.F

Factorisation method:

Express each number as the product of primes and take the product of the least powers of **common** factors to get the H.C.F.

Find the H.C.F. of 2, 4, 6, 8, 10.

Factorise all the numbers into their prime factors 2, 2^2, 2×3, 2^3, 2×5 and choose the common factors with lowest powers. $\therefore$ H.C.F = 2. Remember that 1 is another factor which divides all the given nos. but 2 is the largest possible in the given case. In fact 1 will be a factor in all the cases but we are concerned with the largest possible.

Division Method:

Divide the larger number by smaller one. Now divide the divisor by the remainder. Repeat the process of dividing the preceding divisor by the remainder last obtained, till the remainder zero is obtained. The last divisor is the required H.C.F.

Find the H.C.F of 24 and 36.

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24) 36 (1
    24
    12 ) 24 ( 2
         24
          x
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$\therefore$ H.C.F of the given numbers = 12.

Toolkit

i) Product of two nos. = L.C.M. x H.C.F.

ii) Product of n nos. = L.C.M of n numbers. x (H.C.F. of each pair)$^{n-1}$

iii) H.C.F. of given numbers always divides their L.C.M.

iv) L.C.M. of a series of fractions = $\dfrac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$

V) H.C.F. of a series of fractions = $\dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$

TIP

Always reduce the fractions into their lowest or simplest forms before finding their HCF or LCM.

Co- Prime Numbers

Two numbers are said to be co- prime if their H.C.F is 1 i.e. the divisor is 1 when the remainder is 0.

e.g. (2, 3). These two nos. may or may not be always prime. **e.g.** (4, 9).

PRACTICAL APPLICATIONS OF L.C.M. & H.C.F.:

Example

A bell tolls after every 2 s. A second bell tolls after every 3s, A third every 4s, a fourth every 10s, When will all the bells toll together?

Sol. All the bells will toll together after the time, which is divisible by 2, 3, 4 and 10. Therefore time must be equal to L.CM. of 2, 3, 4, 10.

2	2-3-4-10
2	1-3-2-5
	1-3-1-5

$\therefore$ L.C.M. = 60 s.

Hence 4 bells will toll together after every 60s.

Example

Find the greatest no that will divide 55, 127 & 175 so as to leave the same remainder in each case.

Sol. As 55, 127, 175 leave the same remainder when divided by certain number.

$\therefore$ Difference = 127 – 55, 175 – 127, 175 – 55 must be divisible by that number.

$\therefore$ Greatest number = H.C.F. of (127 – 55), (175 – 127), (175 – 55)

= HCF of 72, 48, 120

By factorisation method :

$72 = 2^3 \times 3^2$

$48 = 2^4 \times 3$

$120 = 2^3 \times 3 \times 5$

HCF of 72, 48, 120 = $2^3 \times 3 = 24$.

Hence the greatest no that will divide 55, 127 & 175 leaving same remainder = 24.

> ***Do You know ?***
> When 2 or more numbers divided by a certain number leave the same remainder then the differences of the given nos. are always divisible by that number.

H.C.F. & L.C.M. of Decimals

In given numbers, make the same number of places of decimals by supplying zeros in some numbers if necessary. Considering these numbers without decimal point, find H.C.F. OR L.C.M. as the case may be. Now, in the result mark off as many decimal places as there are in each of the numbers.

Example

Find the H.C.F & L.C.M. of 1.8, 5.4 and 12.

Sol. Making the same place of decimals, the numbers may be written as 1.8, 5.4 and 12.0.

Without decimals they are 18, 54, 120.

H.C.F. of 18, 54, 120 is 6.

$\therefore$ H.C.F. of 1.8, 5.4 & 12 is 0.6.

Again L.C.M. of 18, 54 & 120 is 1080.

$\therefore$ L.C.M. of 1.8, 5.4, 12 is 108.0, i.e. 108.

Important Results

IMPORTANT RESULTS

S.No.	Type of Problem	Approach of Problem
1.	Find the GREATEST NUMBER that will *exactly* divide x, y, z.	Required number = H.C.F. of x, y, and z (greatest divisor).
2.	Find the GREATEST NUMBER that will divide x, y and z leaving remainders a, b and c respectively.	Required number (greatest divisor) = H.C.F. of (x – a), (y – b) and (z – c).
3.	Find the LEAST NUMBER which is *exactly* divisible by x, y and z.	Required number = L.C.M. of x, y and z (least divided).
4.	Find the LEAST NUMBER which when divided by x, y and z leaves the remainders a, b and c respectively.	Then, it is always observed that (x- a) = (z – b) = (z – c) = K (say). $\therefore$ Required number = (L.C.M. of x, y and z) – (K).
5.	Find the LEAST NUMBER which when divided by x, y and z leaves the same remainder 'r' each case.	Required number = (L.C.M. of x, y and z) + r.
6.	Find the GREATEST NUMBER that will divide x, y and z leaving the same remainder in each case.	Required number = H.C.F. of (x – y), (y – z) and (z – x).
7.	Find the H.C.F. of $\frac{x}{y}$, $\frac{a}{b}$ and $\frac{m}{n}$.	H.C.F of fractions $= \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$.
8.	Find the L.C.M. of $\frac{x}{y}$, $\frac{a}{b}$ and $\frac{m}{n}$.	L.C.M. of fractions $= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$.
9.	Find the H.C.F. of decimal numbers.	**Step 1:** Find the H.C.F. of the given numbers **without** decimal. **Step 2:** Put the decimal point (in the H.C.F. of Step 1) from right to left according to the MAXIMUM decimal places among the given numbers.
10.	Find the L.C.M. of decimal numbers.	**Step 1:** Find the L.C.M. of the given numbers **without** decimal. **Step 2:** Put the decimal point (in the L.C.M. of Step 1) from right to left at the place equal to the MINIMUM decimal places among the given numbers.